Spin rotation for ballistic electron transmission induced by spin-orbit interaction

Evgeny N. Bulgakov [1] and Almas. F. Sadreev,[1,2]
[1] *Institute of Physics, Academy of Sciences, 660036 Krasnoyarsk, Russia,* [2] *Department of Physics and Measurement Technology, Linköping University, S-581 83 Linköping, Sweden*

We study spin dependent electron transmission through one- and two-dimensional curved waveguides and quantum dots with account of spin-orbit interaction. We prove that for a transmission through arbitrary structure there is no spin polarization provided that electron transmits in isolated energy subband and only two leads are attached to the structure. In particular there is no spin polarization in the one-dimensional wire for which spin dependent solution is found analytically. The solution demonstrates spin evolution as dependent on a length of wire. Numerical solution for transmission of electrons through the two-dimensional curved waveguides coincides with the solution for the one-dimensional wire if the energy of electron is within the first energy subband. In the vicinity of edges of the energy subbands there are sharp anomalies of spin flipping.

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I. INTRODUCTION

The electron spin precession phenomena at zero magnetic field induced by a variable spin-orbit interaction (SOI) in 2DEG systems was firstly proposed by Datta and Das [1] as a way for the realization of the spin transistor. For this, the spin precession is controlled via the Razhba SOI associated with the interface electric field present in the GaAs heterostructures that contains the 2DEG channel [2]

$$V_{SO}^{\alpha} = \hbar\alpha[\hat{p}_x\sigma_y - \hat{p}_y\sigma_x]. \tag{1}$$

The reason of spin precession is that the spin operators do not commutate with the SOI operator what leads to the spin evolution for electron transport. In particular the SOI has a polarization effect on particle scattering processes [3] and this effect was considered for different geometries of confinement of the 2DEG [4, 5, 6, 7, 8, 9].

The most simple case of the stripe geometry with the x-axis along the stripe and the z-axis perpendicular to the stripe gives the following transformation of spin state after transmission

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix} \tag{2}$$

where [1, 9]

$$\theta = 2m^*\alpha L \tag{3}$$

and L is a length of the stripe. Therefore, the Razhba SOI induces a spin precession of the transmitted electrons. Notice that the spin precession is energy independent. This result is valid if the confinement energy $\hbar^2/2m^*d^2$, where d is a width of the stripe, is much larger than the spin-splitting energy induced by the SOI, and therefore, the intersubband mixing is negligible [9]. For the strong SOI the spin rotation angle θ becomes to be the Fermi energy depend for ballistic transport of electrons in the quasi one-dimensional wires and stripes [6, 9]. Whatever the Razhba SOI leads to the spin precession in the (x, z) plane. Here we consider similar phenomena for electron transmission through the curved waveguide and quantum dots. The main difference between the straight waveguide and curved one is that the spin rotation is given by two angles.

Next, we find out conditions under which there is no a spin polarization of transmitted electrons. We imply that a flow of incident electrons have no spin polarization. By the spin polarization we consider the mean spin $< \sigma_\alpha >, \alpha = x, y, z$ averaged over the electron flow. In particular for the transmission through quantum dot we show a principal role of the third lead for the spin polarization.

II. THE SPIN-ORBIT INTERACTION IN THE INHOMOGENEOUS TWO-DIMENSIONAL CASE

We write the total Hamiltonian of a confined 2DEG as

$$H = -\frac{\hbar^2}{2m^*}\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}\right) + V(x, y) + V_{SO} \tag{4}$$

where $V(x, y)$ is the lateral confining potential. Following to Moroz and Barnes [6] we assume that the SOI operator V_{SO} is formed by three contributions

$$V_{SO} = V_{SO}^{\alpha} + V_{SO}^{\gamma} + V_{SO}^{\alpha\alpha}.$$

The first V_{SO}^{α} is related to the Razhba SOI (1), in which the SOI constant α proportional to the macroscopic interface-induced electric field is considered as constant. The second contribution V_{SO}^{γ} to the SOI comes from the electric field $\mathbf{E}(x, y)$ related to the confining potential.

In order to derive the second contribution to the SOI we begin with general description of SOI [10]

$$V_{SO} = -\frac{e}{4m^2c^2}\left\{\sigma(\mathbf{E}\times\hat{\mathbf{p}}) + \frac{i\hbar}{2}\sigma(\nabla\times\mathbf{E})\right\}. \tag{5}$$

For microscopic electric field $\mathbf{E}$ the second term in (5) equals zero. However for model cases of the confining potential $V(x,y)$ the electric field can violate an equality $\nabla\times\mathbf{E}=0$. In this case the second term in (5) is necessary to provide a hermiticity of the total SOI operator.

For a 2DEG confined at semiconductor heterostructure interface we can reduce the z-coordinate performing average over electron wave function $\psi_0(z)$ strongly localized along the z-direction

$$V_{SO} \Rightarrow \int dz\psi_0(z)V_{SO}\psi_0(z). \qquad (6)$$

As a result we obtain

$$V_{SO}^{\gamma} = -\gamma\Bigg\{\sigma_z(E_x\hat{p}_y - E_y\hat{p}_x) - E_z(\sigma_x\hat{p}_y - \sigma_y\hat{p}_x)$$

$$-\frac{i}{2}\hbar\sigma_z\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) - \frac{i}{2}\hbar\left(\sigma_y\frac{\partial E_z}{\partial x} - \sigma_x\frac{\partial E_z}{\partial y}\right)\Bigg\}. \qquad (7)$$

Here electric field components are considering in a meaning of integral (6) and depend on x, y only.

For particular case of straight wire directed along the y-axis with the lateral confining potential $U = U(x)$ we obtain from (7) the expression given by Moroz and Barnes (formula (5) in [6]).They used a parabolic approximation for the confining potential. Here we consider a popular hard wall approximation and imply the following confining potential

$$U(x) = \begin{cases} 0, & if \quad |x| < d/2, \\ U_0, & if \quad |x| \geq d/2 \end{cases}.$$

Then substituting the electric field $E_x = -U'(x)$ into (7) we have

$$V_{SO}^{\gamma}(x) = \hbar k\gamma\sigma_z sign(x)U_0\delta(x \mp d/2). \qquad (8)$$

For $|x| > d/2$ we have from the Schrödinger equation the following solution

$$\psi(x) = C\exp\left(-\frac{\sqrt{2m^*(U_0-E)}}{\hbar}x\right) \qquad (9)$$

where C is the normalization constant. Using a property of delta function that a difference between derivatives of the wave function at the right and left of the delta function is obeying to $\Delta\psi'(\pm d/2)) = \pm 2m^*k\sigma_z\gamma U_0\psi(\pm d/2)$ we have from (9) that

$$\Delta\psi'(\pm d/2)) \to 0$$

for $U_0 \to \infty$. Therefore in the hard wall approximation an effect of the second contribution V_{SO}^{γ} limits to zero.

Next, for numerical computation of the transmission through the semiconductor heterostructure we assume a connection at least to two electrodes in which there is no the SOI. Then we can specify the electron state by quantum numbers, the number of the energy subband n and spin projection $\sigma = \sigma_z$. This assumption implies that far from waveguides or quantum dots the SOI constant α equals zero in the electrodes. Neglecting by real space behavior of the microscopic electric field at the edge of the heterestructure we assume that the field is directed normal to the plane of the heterostructure everywhere and has a stepwise behavior at the edges. As a result we obtain the stepwise behavior for the Razhba SOI constant α. Such a model was used by Hu and Matsuyama [11]. Similar to (7) we obtain that the third contribution to the SOI takes the following form

$$V_{SO}^{\alpha\alpha} = -\hbar^2\frac{i}{2}\left(\sigma_y\frac{\partial\alpha}{\partial x} - \sigma_x\frac{\partial\alpha}{\partial y}\right). \qquad (10)$$

III. THE TRANSMISSION THROUGH BILLIARD WITH THE SOI

In this section we prove that the SOI gives no spin polarization for electron transmission through arbitrary billiards if energy of incident electron belongs to the first energy subband. In dimensionless form the stationary Schrödinger equation has the following form

$$\left\{-\nabla^2 + v_{SO}\right\}\psi = \epsilon\psi, \quad \psi = \begin{pmatrix} u_1(x,y) \\ u_2(x,y)\end{pmatrix}. \qquad (11)$$

Here $\epsilon = E/E_0, E_0 = \frac{\hbar^2}{2m^*L^2}$,

$$v_{SO} = \beta\left(i\sigma_x\frac{\partial}{\partial y} - i\sigma_y\frac{\partial}{\partial x}\right) - \frac{i}{2}\left(\sigma_y\frac{\partial\beta}{\partial x} - \sigma_x\frac{\partial\beta}{\partial y}\right), \qquad (12)$$

L is a characteristic scale of the system, $\beta = 2m^*\alpha L$ is the dimensionless SOI constant. Correspondingly in Eqs (11) and (12) coordinates x, y are also dimensionless.

Let S be an area of the structure under consideration which involves a billiard the SOI and leads as shown in Fig. 1.

Let Γ denote a boundary which crosses input lead and output one at Γ_1 and Γ_2 respectively. We suppose that there is no spin-orbit interaction in the leads, *i.e* $\beta = 0$ at $\Gamma_i, i = 1,2$. At the rest of boundary Γ we imply the Dirichlet boundary conditions for solution of Schrödinger equation (11) $\psi|_{\Gamma} = 0$. As the scale L we take $L = d$.

Therefore we can write solution in the electrodes as follows

$$|inc, n, \sigma >= \sqrt{2}\sin(\pi ny)\exp(ik_nx)|\sigma>,$$
$$|refl, n, \sigma >= \sqrt{2}\sum_{m,\sigma'} r_{mn,\sigma,\sigma'}\sin(\pi my)\exp(-ik_mx)|\sigma'>,$$
$$|tr, n, \sigma >= \sqrt{2}\sum_{m,\sigma'} t_{mn,\sigma,\sigma'}\sin(\pi my')\exp(ik_mx')|\sigma'> \qquad (13)$$

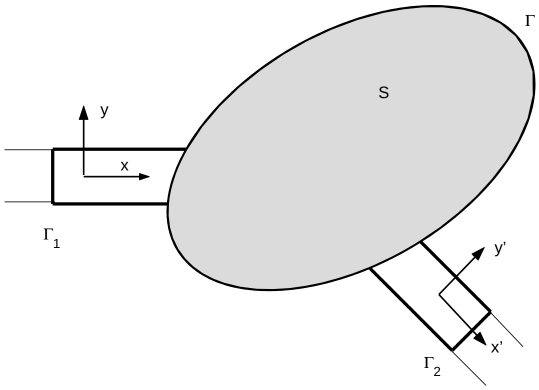

FIG. 1: Schematical view of two-dimensional billiard with two attached leads. Dashed area shows a region with the SOI. The area S has boundary Γ which crosses input lead and output one at Γ_1 and Γ_2 respectively.

where $|\sigma>$ is the spin states defined spin projection along some axis, say, the z-axis. The energy is

$$\epsilon = k_n^2 + \pi^2 n^2 \tag{14}$$

where $n = 1, 2, \dots$ numerates a number of the energy subbands.

Introduce complex derivatives

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) \tag{15}$$

we write the Schrödinger equation (11) as follows

$$\begin{aligned}
&\left(\frac{\partial^2}{\partial z \partial z^*} + \frac{1}{4}\epsilon\right) u_1 + \frac{\beta}{2}\frac{\partial u_2}{\partial z} + \frac{u_2}{4}\frac{\partial \beta}{\partial z} = 0,\\
&\left(\frac{\partial^2}{\partial z \partial z^*} + \frac{1}{4}\epsilon\right) u_2 - \frac{\beta}{2}\frac{\partial u_1}{\partial z^*} - \frac{u_1}{4}\frac{\partial \beta}{\partial z^*} = 0,
\end{aligned} \tag{16}$$

where u_1, u_2 are the components of the spin state. Assume that there is auxiliary degenerated state with components v_1, v_2. In particular, it might be the Kramers degenerated state. Then, for these two states the Green formula follows

$$\int_S (u\Delta v - v\Delta u) dS = \oint_\Gamma \left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right) dl \tag{17}$$

where n is an exterior normal to the boundary Γ.

From the Schrödinger equation we have

$$\begin{aligned}
&v_2\left(\frac{\partial^2}{\partial z \partial z^*} + \frac{1}{4}\epsilon\right) u_1 + \frac{\beta}{2}v_2\frac{\partial u_2}{\partial z} + \frac{1}{4}u_2 v_2\frac{\partial \beta}{\partial z} = 0,\\
&u_2\left(\frac{\partial^2}{\partial z \partial z^*} + \frac{1}{4}\epsilon\right) v_1 + \frac{\beta}{2}u_2\frac{\partial v_2}{\partial z} + \frac{1}{4}u_2 v_2\frac{\partial \beta}{\partial z} = 0,\\
&u_1\left(\frac{\partial^2}{\partial z \partial z^*} + \frac{1}{4}\epsilon\right) v_2 - \frac{\beta}{2}u_1\frac{\partial v_1}{\partial z^*} - \frac{1}{4}u_1 v_1\frac{\partial \beta}{\partial z^*} = 0,\\
&v_1\left(\frac{\partial^2}{\partial z \partial z^*} + \frac{1}{4}\epsilon\right) u_2 - \frac{\beta}{2}v_1\frac{\partial u_1}{\partial z^*} - \frac{1}{4}u_1 v_1\frac{\partial \beta}{\partial z^*} = 0.
\end{aligned} \tag{18}$$

Combining each couple of equations in (18) we obtain

$$\begin{aligned}
&v_2\frac{\partial^2 u_1}{\partial z \partial z^*} + u_2\frac{\partial^2 v_1}{\partial z \partial z^*} + \frac{1}{4}\epsilon(u_1 v_2 + u_2 v_1) + \frac{1}{2}\frac{\partial(\beta u_2 v_2)}{\partial z} = 0,\\
&u_1\frac{\partial^2 v_2}{\partial z \partial z^*} + v_1\frac{\partial^2 u_2}{\partial z \partial z^*} + \frac{1}{4}\epsilon(u_1 v_2 + u_2 v_1) - \frac{1}{2}\frac{\partial(\beta u_1 v_1)}{\partial z^*} = 0.
\end{aligned} \tag{19}$$

Extracting the second equation from the first one in (19) we obtain

$$\begin{aligned}
&v_2\frac{\partial^2 u_1}{\partial z \partial z^*} - u_1\frac{\partial^2 v_2}{\partial z \partial z^*} + u_2\frac{\partial^2 v_1}{\partial z \partial z^*} - v_1\frac{\partial^2 u_2}{\partial z \partial z^*}\\
&+ \frac{1}{2}\frac{\partial(\beta u_1 v_1)}{\partial z^*} + \frac{1}{2}\frac{\partial(\beta u_2 v_2)}{\partial z} = 0.
\end{aligned} \tag{20}$$

Integration of this equation over the billiard area S with use of the Green formula (17) gives the following

$$\begin{aligned}
&\oint_\Gamma \left(v_2\frac{\partial u_1}{\partial n} - u_1\frac{\partial v_2}{\partial n}\right) dl + \oint_\Gamma \left(u_2\frac{\partial v_1}{\partial n} - v_1\frac{\partial u_2}{\partial n}\right) dl\\
&+ \int_S \frac{\partial(2\beta u_1 v_1)}{\partial z^*} dS + \int_S \frac{\partial(2\beta u_2 v_2)}{\partial z} dS = 0.
\end{aligned} \tag{21}$$

Since at Γ either $u_1 = 0, v_1 = 0$, or $\beta = 0$, the last two integrals in (21) equal zero and formula (21) can be rewritten as follows

$$\begin{aligned}
&\sum_{i=1,2}\int_{\Gamma_i} \left(v_2\frac{\partial u_1}{\partial n} - u_1\frac{\partial v_2}{\partial n}\right) dl\\
&+ \sum_{i=1,2}\int_{\Gamma_i} \left(u_2\frac{\partial v_1}{\partial n} - v_1\frac{\partial u_2}{\partial n}\right) dl = 0.
\end{aligned} \tag{22}$$

This formula is sufficient to establish some symmetry rules between ingoing and outgoing states. Let us consider the first-channel transmission for $\epsilon < 4\pi^2$. In order to ignore evanescent modes we will consider that boundaries Γ_i cross the leads far from the scattering region as shown in Fig. 1 . Let electron incidents from the input lead being completely spin polarized up. It means that for the incident state (13) $|\sigma> = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. We denote the corresponding state interior the structure S as $\begin{pmatrix} u_{1\uparrow}(x,y) \\ u_{2\uparrow}(x,y) \end{pmatrix}$ which is used as the u-solution in Eq. (22). Correspondingly $\begin{pmatrix} u_{1\downarrow}(x,y) \\ u_{2\downarrow}(x,y) \end{pmatrix}$ denotes the v-solution in Eq. (22) for the case of electron incidenting with spin polarized down . We suppose that the boundaries Γ_1 and Γ_2 cross the leads normally the leads and the x-axis is parallel to the leads. Hence the normal n is parallel to the x-axis. Then from (13) at the boundary Γ_2 which crosses the output lead we obtain the following relations

$$\frac{\partial f}{\partial n} = ik_1 f \tag{23}$$

where function f refers to all components $u_{1\uparrow}, u_{2\uparrow}, u_{1\downarrow}, u_{2\downarrow}$.

These relations allow to exclude the boundary Γ_2 from (22). At the boundary Γ_1 which crosses the input lead we have

$$\frac{\partial u_{1\uparrow}}{\partial n} = ik_1 u_{1\uparrow} - 2ik_1 \sin(\pi y),$$
$$\frac{\partial u_{2\uparrow}}{\partial n} = ik_1 u_{2\uparrow},$$
$$\frac{\partial u_{1\downarrow}}{\partial n} = ik_1 u_{1\downarrow},$$
$$\frac{\partial u_{2\downarrow}}{\partial n} = ik_1 u_{2\downarrow} - 2ik_1 \sin(\pi y). \quad (24)$$

We imply here that the origin of the x, y coordinate system is at the boundary Γ_1. Substituting the relations (24) into Eq (22) we obtain

$$\int_{\Gamma_1} (u_{1\uparrow} - u_{2\downarrow}) \sin(\pi y)\, dy = 0. \quad (25)$$

Since at the boundary Γ_1

$$u_{1\uparrow} = \tilde{u}_{1\uparrow}(x) \sin(\pi y), u_{2\downarrow} = \tilde{u}_{2\downarrow}(x) \sin(\pi y).$$

we obtain from (25)

$$u_{1\uparrow} = u_{2\downarrow}.$$

Thus from (13) it follows that amplitudes of the reflection

$$r_{\uparrow,\uparrow} = r_{\downarrow,\downarrow}. \quad (26)$$

Next, we take that the state $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_{1\uparrow} \\ u_{2\uparrow} \end{pmatrix}$ coincides with the state $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ in (22). Then equation (22) simplifies as follows

$$\sum_{i=1,2} \int_{\Gamma_i} \left(u_2 \frac{\partial u_1}{\partial n} - u_1 \frac{\partial u_2}{\partial n} \right) dl = 0.$$

Substituting into this formula relations (24) we obtain

$$-2ik \int_{\Gamma_1} u_{2\uparrow} \sin(\pi y)\, dy = 0. \quad (27)$$

It gives us that $u_{2\uparrow} = 0$ or according to (13) $r_{\uparrow\downarrow} = 0$. Also similarly we obtain that $u_{1\downarrow} = 0$ at the boundary Γ_1. Thus we can write the second symmetry rule for reflection amplitudes

$$r_{\uparrow,\downarrow} = r_{\downarrow,\uparrow} = 0. \quad (28)$$

From symmetry rules (26) and (28) and from the current preservation it follows that the transmission probabilities

$$T_\sigma = \sum_{\sigma'} |t_{\sigma,\sigma'}|^2 = T \quad (29)$$

do not depend on the spin polarization of incident electron.

Till now we considered incident waves as spin polarized along the z-axis at the boundary Γ_1. Let now consider a flow of incident electrons which have no averaged spin polarization. In particular we can present that half of electrons have the incident state with spin up and half of electrons have the incident state with spin down. Let us consider corresponding transmitted waves at the boundary Γ_2. We prove that for a transmission through the billiard with two attached leads there is no averaged spin polarization, *i.e.* $< \sigma_\alpha >= 0$, $\alpha = x, y, z$ if electron incidents being spin unpolarized in the first energy subband. As previously we take the incident state in the form (13) and write the states in leads as

$$|\psi_\uparrow >= \begin{pmatrix} u_{1\uparrow} \\ u_{2\uparrow} \end{pmatrix}, \quad |\psi_\downarrow >= \begin{pmatrix} u_{1\downarrow} \\ u_{2\downarrow} \end{pmatrix} \quad (30)$$

where the arrows up and down indicate that electron incident with spins up and down. We take in the Green formulas (22) the first function u as $|\psi_\uparrow >$ and the second function v as $\hat{\sigma}_y \hat{C} |\psi_\downarrow >$ where $\hat{C}$ means a complex conjugation. It means that the second function is the Kramers degenerated state. Hence

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_{1\uparrow} \\ u_{2\uparrow} \end{pmatrix}, \quad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} iu_{2\downarrow}^* \\ -iu_{1\downarrow}^* \end{pmatrix}. \quad (31)$$

Let us calculate integral (22). At a boundary Γ_1 crossing the input lead the integral equals zero since in the input lead $u_2 = 0, v_1 = 0$. The second contribution into integral (22) relates to the boundary Γ_2 crossing the output lead. Using transmitted solution (13) one can write Eq. (22) as follows

$$\int_{\Gamma_2} \left(v_2 \frac{\partial u_1}{\partial n} - u_1 \frac{\partial v_2}{\partial n} \right) dl + \int_{\Gamma_2} \left(u_2 \frac{\partial v_1}{\partial n} - v_1 \frac{\partial u_2}{\partial n} \right) dl$$

$$= 2ik \oint_{\Gamma_2} (v_2 u_1 - u_2 v_1) dy = 0.$$

Therefore $u_1 v_2 = u_2 v_1$, or in terms of notations (31)

$$u_{1\uparrow} u_{1\downarrow}^* = -u_{2\uparrow} u_{2\downarrow}^*. \quad (32)$$

From (32) it obviously follows

$$|u_{1\uparrow}||u_{1\downarrow}| = |u_{2\uparrow}||u_{2\downarrow}|.$$

Moreover relation (29) implies that

$$|u_{1\uparrow}|^2 + |u_{2\uparrow}|^2 = |u_{1\downarrow}|^2 + |u_{2\downarrow}|^2$$

From these two relations one can obtain that

$$|u_{2\uparrow}| = |u_{1\downarrow}|, |u_{1\uparrow}| = |u_{2\downarrow}|. \quad (33)$$

Finally relations (32) and (33) give

$$u_{1\uparrow} u_{2\uparrow}^* = -u_{1\downarrow} u_{2\downarrow}^*. \quad (34)$$

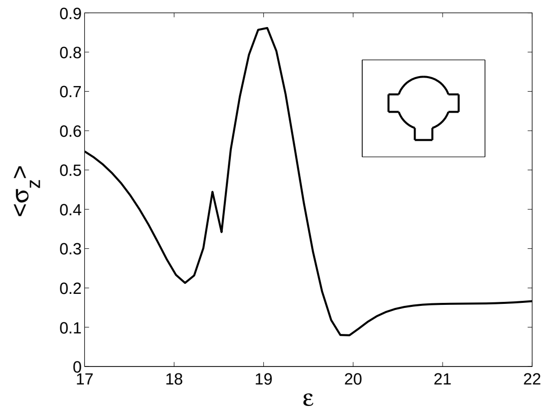

FIG. 2: Spin polarization of electrons transmitted through the three terminal quantum dot versus energy of electron in the first energy subband. An inset above shows a geometry of the structure.

Mean values of spin components in corresponding states (30) are the following

$$< \sigma_x >_\uparrow = Re(u_{1\uparrow} u^*_{2\uparrow}), < \sigma_y >_\uparrow = Im(u_{1\uparrow} u^*_{2\uparrow}),$$
$$< \sigma_z >_\uparrow = |u_{1\uparrow}|^2 - |u_{2\uparrow}|^2,$$
$$< \sigma_x >_\downarrow = Re(u_{1\downarrow} u^*_{2\downarrow}), < \sigma_y >_\downarrow = Im(u_{1\downarrow} u^*_{2\downarrow}),$$
$$< \sigma_z >_\downarrow = |u_{1\downarrow}|^2 - |u_{2\downarrow}|^\cdot \quad (35)$$

Eqs (33) - (35) give rise to

$$< \sigma_\alpha >_\uparrow = - < \sigma_\alpha >_\downarrow, \, \alpha = x, y, z, \quad (36)$$

i.e. the spin polarizations are exactly opposite in sign for transmission of electrons incidenting in corresponding spin polarized states.

Thus, for the transmission through any billiard with the SOI with two attached leads the spin polarization does not exist if the flow of electrons incidents in the first energy subband and have no spin polarization. Also, if there is no intersubband transmissions $t_{mn,\sigma\sigma'} = 0, m \neq n$, the spin polarization equals zero for arbitrary energy. It takes place approximately, for example, for adiabatic structures similar to curved waveguides (section V). However in a vicinity of edges of the energy subbands $\pi^2 n^2$ the SOI gives rise to intersubband mixing. As a result we obtain in numerical calculations strong spin polarization near the edges. Moreover, if the billiard is connected to three or more leads, the spin polarization of transmitted electrons exists even for the transmission in the first energy subband. The effect of the third lead is demonstrated in Fig. 2. Hence this effect propose a way of the spin transistor complimentary to the way proposed by Datta and Das [1]. The spin polarization of transmitted electrons can be governed by a value of connection of the third lead with the quantum dot. The most simple way is to apply local electric field in the vicinity of the connection which implies potential barrier closing the connection of the dot with the third lead.

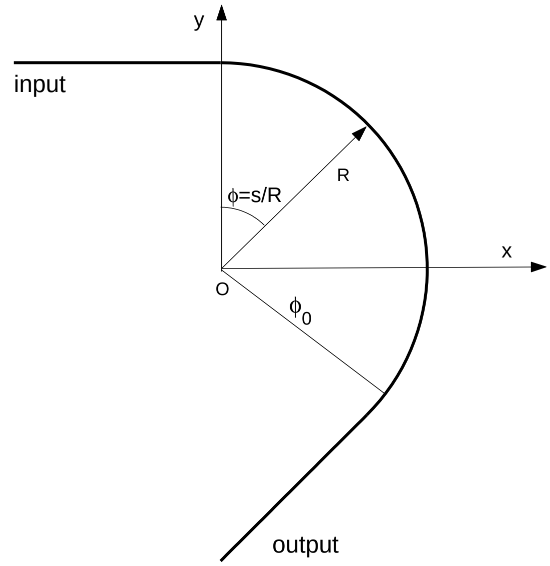

FIG. 3: Schematical view of one-dimensional curved wire.

IV. THE ONE DIMENSIONAL CURVED WIRE

A model in which only the single channel transmission takes place is the one-dimensional wire. Therefore for a transmission through the one-dimensional wire of any form the SOI can not give rise to the spin polarization. However this model is interesting by that allows to find spin evolution analytically. A case of straight wire was considered by [1, 9]. Here we consider a curved wire consisted of a segment of circle with radius R attached to infinite straight one dimensional wires as shown in Fig. 3.

We take a length of the segment as $L = \phi_0 R$ and a position coordinate as $s = \phi R$. The Hamiltonian of the wire has the following form [12, 13]

$$H = \frac{\hbar^2}{2m^* R^2} \widetilde{H},$$

$$\widetilde{H} = \left[\frac{\partial}{i \partial \phi} + \frac{\beta}{2} \left(\sigma_y \cos\phi + \sigma_x \sin\phi \right) \right]^2 - \frac{\beta^2}{4}, \quad (37)$$

where $\beta = 2m^* \alpha R$ is the dimensionless SOI constant. Since $[J_z, H] = 0$ where $J_z = -i\frac{\partial}{\partial\phi} - \frac{1}{2}\sigma_z$, a particular solution of the stationary Shcrödinger equation $\widetilde{H}|\psi> = \epsilon|\psi>$ has the following form [12, 13, 14]

$$|\psi> = \begin{pmatrix} A e^{i\mu\phi} \\ B e^{i(\mu-1)\phi} \end{pmatrix}. \quad (38)$$

The parameter μ defines the dimensionless wave number as $k = \mu/R$ and is arbitrary until the boundary conditions are imposed. Substituting the state (38) into the Shcrödinger equation one can obtain the following relation between the energy of electron ϵ and the wave number μ

$$(\epsilon - \mu^2)(\epsilon - (\mu - 1)^2) - \beta^2(\mu - 1/2)^2 = 0 \quad (39)$$

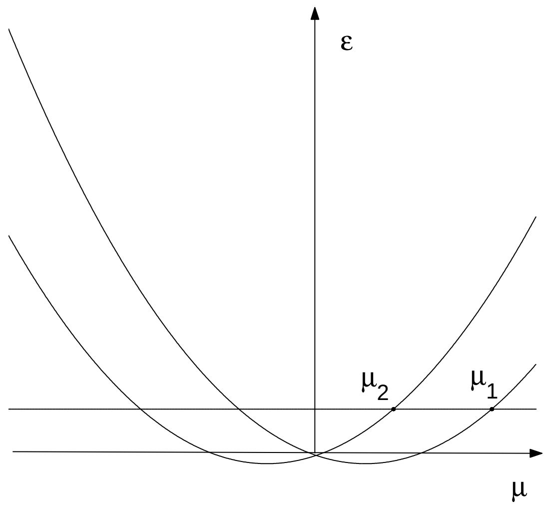

FIG. 4: The energy spectrum defined by formula (40) for $\alpha = 1$. Values of μ corresponded to clockwise movement of electron along the curved wire are shown by thick points.

which gives

$$\epsilon_\nu = (\mu - 1/2)^2 + 1/4 + \nu|\mu - 1/2|\sqrt{\beta^2 + 1/4}, \quad \nu = \pm 1. \tag{40}$$

The spectrum (40) is shown in Fig. 4.

For fixed energy ϵ Eq.(40) gives four solutions for the wave number μ. It is well known that [3] for electron transmission through potential profile a reflection is negligibly small if the characteristic length of inhomogeneity much exceeds the wave length (adiabatic regime). For our case we assume that the radius of curvature of the wire is much larger in comparison with the electron wave length. So we can ignore the reflection for electron transmission through the quasi one-dimensional waveguide.

Since there is no reflection for transmission through the one-dimensional waveguide we need only those values of the wave number μ which correspond to clockwise movement of electron in the waveguide. We denote its as μ_1, μ_2 shown in Fig. 4 . In what follows we use the following relation between μ_1 and μ_2:

$$2\lambda = \mu_2 - \mu_1 = -\sqrt{1 + \beta^2}. \tag{41}$$

Therefore general solution of the Shcrödinger equation for the electron transmission without reflection can be written as follows

$$|\psi(\phi) >= \sum_{\nu=1,2} a_\nu e^{i(\mu_\nu - 1/2)\phi} U(\phi) \begin{pmatrix} A_\nu \\ B_\nu \end{pmatrix} \tag{42}$$

where

$$\begin{pmatrix} 1 - \sqrt{1+\beta^2} & -i\beta \\ i\beta & -1 - \sqrt{1+\beta^2} \end{pmatrix} \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = 0, \tag{43}$$

$$\begin{pmatrix} 1 + \sqrt{1+\beta^2} & -i\beta \\ i\beta & -1 + \sqrt{1+\beta^2} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = 0. \tag{44}$$

The matrix $U(\phi)$ has the following form

$$U(\phi) = \begin{pmatrix} \exp(i\phi/2) & 0 \\ 0 & \exp(-i\phi/2) \end{pmatrix}. \tag{45}$$

Evolution of the electron state (42) as length $s = \phi R$ of the curved wire can be presented as

$$|\psi(\phi) >= e^{i(\mu_1 + \mu_2 - 1)\phi/2} U(\phi) \Lambda$$

$$\begin{pmatrix} \exp(i\lambda\phi) & 0 \\ 0 & \exp(-i\lambda\phi) \end{pmatrix} \Lambda^{-1} |\psi(0) > \tag{46}$$

where

$$\Lambda = \begin{pmatrix} A_2 & A_1 \\ B_2 & B_1 \end{pmatrix}. \tag{47}$$

From Eqs (43) and (44) we can rewrite (47) as follows

$$\Lambda = \begin{pmatrix} A_2 & -B_2^* \\ B_2 & A_2^* \end{pmatrix} \tag{48}$$

and

$$\frac{A_2}{B_2} = \frac{i\beta}{(1 - \sqrt{1+\beta^2})}. \tag{49}$$

Eq. (46) can be presented as $|\psi(\phi) >= T(\phi)|\psi(0) >$ which shows that the unitary matrix $T(\phi)$ has a meaning of the transfer one. Since the state $|\psi(\phi) >$ is spinor one the matrix $T(\phi)$ corresponds to rotation matrix for transport of electron along the wire. In general case the rotation matrix is given by the Euler angles $(\varphi, \theta, \gamma)$ [15]

$$R(\varphi, \theta, \gamma) = e^{-i\varphi\frac{1}{2}\sigma_z} e^{-i\theta\frac{1}{2}\sigma_y} e^{-i\gamma\frac{1}{2}\sigma_z}. \tag{50}$$

The rotation matrix R has the following form

$$R(\varphi, \theta, \gamma)$$

$$= \begin{pmatrix} \exp(-i\frac{1}{2}(\varphi+\gamma))\cos(\frac{1}{2}\theta) & -\exp(-\frac{1}{2}i(\varphi-\gamma))\sin(\frac{1}{2}\theta) \\ \exp(i\frac{1}{2}(\varphi-\gamma))\sin(\frac{1}{2}\theta) & \exp(i\frac{1}{2}(\varphi+\gamma))\cos(\frac{1}{2}\theta) \end{pmatrix}. \tag{51}$$

In order to find the Euler angles let us consider to which rotation corresponds matrix

$$\Lambda \begin{pmatrix} \exp(i\lambda\phi) & 0 \\ 0 & \exp(-i\lambda\phi) \end{pmatrix} \Lambda^{-1}. \tag{52}$$

If the matrix Λ were unit, the matrix

$$\begin{pmatrix} \exp(i\lambda\phi) & 0 \\ 0 & \exp(-i\lambda\phi) \end{pmatrix}$$

would correspond to the rotation by the angle $\theta = -2\lambda\phi$ around the z-axis. The matrix Λ in (52) gives rise to the clockwise rotation around the x-axis by the angle θ which satisfies to the following equation

$$\frac{A_2}{B_2} = \cot(\theta/2)e^{-i\varphi} = \frac{i\beta}{1 - \sqrt{1+\beta^2}}. \tag{53}$$

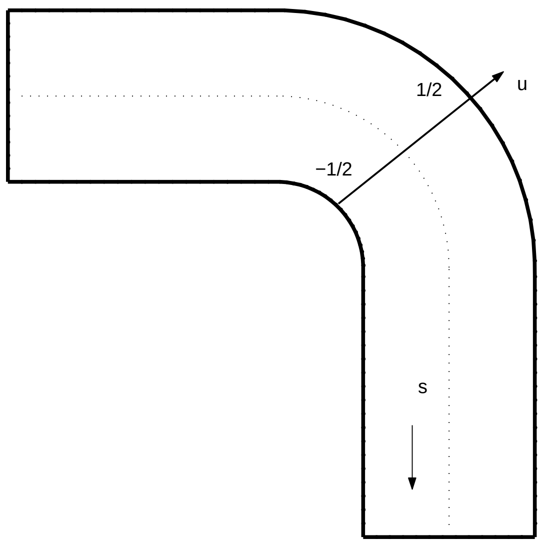

FIG. 5: A fragment of two-dimensional curved wire with width $d = 1$.

In order to fulfill this equation we choose

$$\varphi = \pi/2, \quad \cot\frac{\theta}{2} = \frac{\beta}{\sqrt{1+\beta^2}-1}. \tag{54}$$

The angle γ is remaining undefined. Below we put $\gamma = 0$. Let us choose new axis z' in the (y,z) plane with the angle θ as the angle between the z-axis and z'-axis. Thus, the full rotation matrix consists of the anticlockwise rotation around the the z'-axis by the angle $\phi\sqrt{1+\beta^2}$ and the clockwise rotation by the angle ϕ around the z-axis. The last statement follows from the matrix $U(\phi)$ in Eq.(46). A knowledge of the evolution of the spin state (46) as dependent on the length ϕR of the curved waveguide gives us a possibility to calculate in particular evolution of the spin components. The result of calculation is shown in Fig. 6 by squares, triangles and circles for spin components $\sigma_z, \sigma_x, \sigma_y$ respectively. As seen the results of numerical computation for the two-dimensional curved waveguide are surprisingly close to present one-dimensional model.

V. THE TWO-DIMENSIONAL CURVED WAVEGUIDE

For consideration of the two-dimensional curved waveguide we introduce the curved coordinate system (s, u) [16, 17] where s is the coordinate of central line along of the waveguide shown in Fig. 5.

We express the Hamiltonian of the waveguide in dimensionless form by following way

$$H = \frac{\hbar^2}{2m^*d^2}(\widetilde{H}_0 + v_{SO}),$$

where

$$\widetilde{H}_0 = -\Delta = -g^{-1/2}\frac{\partial}{\partial s}g^{-1/2}\frac{\partial}{\partial s} - g^{-1/2}\frac{\partial}{\partial u}g^{1/2}\frac{\partial}{\partial u}, \tag{55}$$

and d is a width of the waveguide. In what follows we consider a segment of the two-dimensional ring with constant curvature $\gamma = 1/R$ attached to straight leads with the same width as shown in Fig. 5. Therefore for the segment we can write

$$\begin{aligned} x &= a(s) - ub'(s) \\ y &= b(s) + ua'(s) \\ a(s) = -R\cos(s/R), b(s) &= R\sin(s/R) \\ g^{1/2} = 1 + u\gamma(s) &= \frac{u+R}{R} \end{aligned} \tag{56}$$

with $\gamma(s)$ as a curvature of the curved waveguide which is taken below constant. The SOI takes the following form at the curved part of the waveguide

$$v_{SL} =$$

$$\beta\begin{pmatrix} 0 & e^{is/R}\left(\frac{\partial}{\partial u} + ig^{-1/2}\frac{\partial}{\partial s}\right) \\ -e^{-is/R}\left(\frac{\partial}{\partial u} - ig^{-1/2}\frac{\partial}{\partial s}\right) & 0 \end{pmatrix}. \tag{57}$$

At the leads we assume that there is no the spin-orbital interaction ($\beta = 0$) as well as $\gamma = 0, g^{1/2} = 1$.

The Shcrödinger equation

$$\tilde{H}\begin{pmatrix}\psi_\uparrow \\ \psi_\downarrow\end{pmatrix} = \epsilon\begin{pmatrix}\psi_\uparrow \\ \psi_\downarrow\end{pmatrix}$$

with the total Hamiltonian as $\tilde{H} = \tilde{H}_0 + v_{SO}$ takes the following form

$$g^{-1/2}\frac{\partial}{\partial s}\left(g^{-1/2}\frac{\partial\psi_\uparrow}{\partial s}\right) + g^{-1/2}\frac{\partial}{\partial u}\left(g^{1/2}\frac{\partial\psi_\uparrow}{\partial u}\right)$$

$$+\epsilon\psi_\uparrow - \beta e^{is/R}\left(\frac{\partial\psi_\downarrow}{\partial u} + ig^{-1/2}\frac{\partial\psi_\downarrow}{\partial s}\right) = 0,$$

$$g^{-1/2}\frac{\partial}{\partial s}\left(g^{-1/2}\frac{\partial\psi_\downarrow}{\partial s}\right) + g^{-1/2}\frac{\partial}{\partial u}\left(g^{1/2}\frac{\partial\psi_\downarrow}{\partial u}\right)$$

$$+\epsilon\psi_\downarrow + \beta e^{-is/R}\left(\frac{\partial\psi_\uparrow}{\partial u} - ig^{-1/2}\frac{\partial\psi_\uparrow}{\partial s}\right) = 0, \tag{58}$$

The solutions of Eqs (58) which satisfy to the Dirichlet boundary conditions ($u = \pm 1/2$) can be presented as [16, 17]

$$\begin{aligned} \psi_\uparrow(u,s) &= \sum_{n=1}^{\infty} A_{\uparrow n}(s)\sin(\pi n(u+1/2)) \\ \psi_\downarrow(u,s) &= \sum_{n=1}^{\infty} A_{\downarrow n}(s)\sin(\pi n(u+1/2)). \end{aligned} \tag{59}$$

Substitution of (59) into Eqs (58) gives

$$\sum_{n=1}^{\infty}[L_{mn}A''_{\uparrow n}(s)+P_{mn}A_{\uparrow n}(s)-\beta e^{i\gamma s}Q_{mn}A_{\downarrow n}(s)$$

$$-i\beta e^{i\gamma s}R_{mn}A'_{\downarrow n}(s)]=[(\pi m)^2-\epsilon]A_{\uparrow m}$$

$$\sum_{n=1}^{\infty}[L_{mn}A''_{\downarrow n}(s)+P_{mn}A_{\downarrow n}(s)+\beta e^{-i\gamma s}Q_{mn}A_{\uparrow n}(s)$$

$$-i\beta e^{-i\gamma s}R_{mn}A'_{\uparrow n}(s)]=[(\pi m)^2-\epsilon]A_{\downarrow m}. \quad (60)$$

Here we introduced the following notations

$$L_{mn}=2\int_{-1/2}^{1/2}\frac{\sin(\pi m(u+1/2))\sin(\pi n(u+1/2))}{(1+u\gamma)^2}du,$$
$$P_{mn}=2\pi n\int_{-1/2}^{1/2}\frac{\gamma\sin(\pi m(u+1/2))\cos(\pi n(u+1/2))}{1+u\gamma}du,$$
$$R_{mn}=2\int_{-1/2}^{1/2}\frac{\sin(\pi m(u+1/2))\sin(\pi n(u+1/2))}{1+u\gamma}du,$$
$$Q_{mn}=2\pi n\int_{-1/2}^{1/2}\sin(\pi m(u+1/2))\cos(\pi n(u+1/2))du. \quad (61)$$

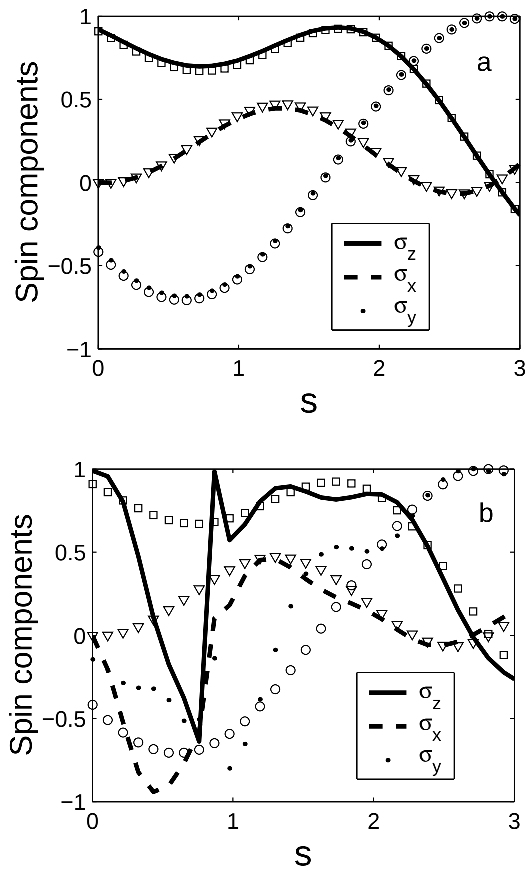

FIG. 6: The spin components as dependent on the length s of a curved two-dimensional waveguide. The result of calculation based on the state (46) for the curved one-dimensional wire is shown by squares (σ_z), triangles (σ_x) and circles (σ_y). The radius of wire $R = d$ where d is the width of waveguide. The dimensionless spin-orbit constant $\beta = 2m^*\alpha d$ equals unit.(a) The dimensionless energy $\epsilon = 25$ (the first channel transmission) and (b) $\epsilon = 39.25$ (near an edge of the second subband).

VI. NUMERICAL RESULTS

In numerical practice we solve the system of Eqs (60) and (61) taking a finite number of waveguide modes. This number of modes was controlled by the normalization condition that sum of the total reflection probabilities and the total transmission ones is to be equaled to unit. The spin components $<\sigma_\alpha>$ were calculated at the attached outgoing straight electrode in which we assumed there is no the spin-orbit interaction by following formula

$$<\sigma_\beta(s)>=\frac{\int_{-1/2}^{1/2}du<\psi(u,s)|\hat{\sigma_\beta}|\psi(u.s)>}{\int_{-1/2}^{1/2}du<\psi(u,s)|\psi(u.s)>}. \quad (62)$$

In Fig.5 the outgoing electrode as well as incoming one are not shown. Fig. 6 shows evolution of the spin components (62) versus the longitudinal coordinate s.

It is surprising that for energy of incident electron far from the edge of energy subband the spin evolution almost coincides with the one-dimensional curved wire shown in Fig. 6 by squares, triangles and circles. In Fig. 7 (a) the energy dependence of the spin components are shown which demonstrates remarkable phenomenon of spin flipping at the edge of the second energy subband $E_2 = (2\pi)^2 \approx 39.4$. It is interesting that increasing of region with the SOI by increasing of length curved waveguide or increasing of the spin-orbit constant leads to double flipping of electron spin for transmission through the waveguide as shown in Fig. 7 (b) . This phenomenon is a consequence of the intersubband mixing by the SOI as it was discussed in section III.

Therefore one can expect strong deviation of the curved two-dimensional waveguides from the one-dimensional one for the spin evolution near edges of the subbands $\pi^2 n^2$. In fact, one can see from Fig. 7 (b) that for energy of incident electron $E \approx 4\pi^2$ the spin evolution with length of the curved waveguide is strongly deviates from the case of one-dimensional curved wire.

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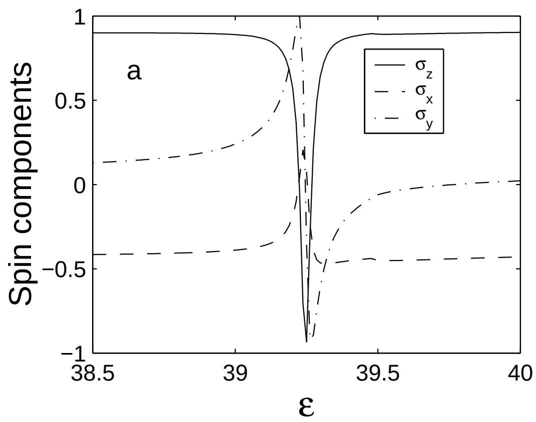
* e-mails:almsa@ifm.liu.se, almas@tnp.krasn.ru



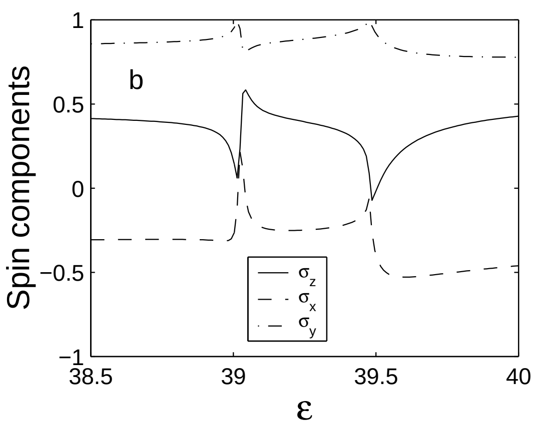


FIG. 7: The spin components as dependent on the energy of incident electron for (a) $\phi_0 = 90^o$-curved waveguide and (b) $\phi_0 = 180^o$ one. $\beta = 1$.

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